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Analytical Prediction of Friction Factors and Heat Transfer Coefficients of Turbulent Forced Convection in Rod Bundles with Surface Roughness

Jian Su†, Mila R. Avelino§ and Atila P. Silva Freire§
†Nuclear Engineering Program (PEN/COPPE/UFRJ),
C.P. 68509, 21945-970 - Rio de Janeiro - Brazil.
§Mechanical Engineering Program (PEM/COPPE/UFRJ),
C.P. 68503, 21945-970 - Rio de Janeiro - Brazil.

Abstract. *In the present work, a simple analytical method is developed for the prediction of the friction factor, f, and of the heat transfer coefficient, C_h, for turbulent flows between rods arranged in hexagonal or square arrays. The cases of flow over a smooth surface and over a rough surface are studied. Different forms of the law of the wall for the velocity and the temperature fields are integrated over the whole flow region to result in algebraic equations for the determination of f and of C_h. The method is applied to arrangements with $P/D < 1.3$.*

Key words: *Rod bundle, turbulent flow, friction factor, heat transfer coefficient, surface roughness.*

1. INTRODUCTION

The flow in channels defined by an external tube and by an internal group of rods arranged in the form of squares or hexagons has many applications in engineering. This geometry, in particular, is typical of the core of a nuclear power reactor. The resulting network of interconnected subchannels, formed between neighbouring fuel rods and between the peripheral fuel rods and the flow tube, gives rise to a complex turbulent flow which has to be well understood and modelled for the design of fuel rod bundles with high performance heat transfer characteristics.

The purpose of this work is to develop a simple analytical method for the prediction of friction factor, f, and the heat transfer coefficient, C_h, for turbulent flows between rods arranged in hexagonal or square arrays. The work provides an extension of the work of Lee(1995) to the thermal case. In addition, the cases of both, flow over a smooth surface and flow over a rough surface are treated. Basically, several forms of the law of the wall are integrated over the entire flow area to yield an algebraic equation for the prediction of both, f and C_h. The laws of the wall take on a classical form but make use of two empirical equations: one for the local variation of the friction velocity and another for the variation of the friction temperature, to assess the behaviour of the flow parameters along the rod periphery.

The present method was applied to infinite bare rod square and triangular arrays with P/D <1.3.

Before moving to the analytical development of the theory, let us first present a short survey of the related literature.

2. SHORT LITERATURE SURVEY

The central importance of the problem under present consideration assures us that, certainly, many studies in this subject were carried out in the past. However, the host of geometrical and physical parameters that are necessary to define the problem makes it very difficult for authors to cover all possible configurations of interest.

A literature survey shows that the first studies concentrated on hexagonal arrays due to their importance in liquid-metal cooled fast breeder reactors (LMFBRs), see, for example, Rehme(1972, 1973), Trupp and Azad(1975), Carajileskov and Todreas(1976), Bartzis and Todreas(1979), Fakory and Todreas(1979). Many aspects of the problem were then studied, taking normally as basis empirical correlation and the hydraulic diameter.

Rehme(1972) conducted investigations on the pressure drop for arrays with rod distance ratios of $P/D = 1.025$–2.324 and Reynolds number in the range Re= 600–200000. These measurements were used in a subsequent paper by the same author(Rehme, 1973) where a simple method for prediction of the friction factor in non-circular channels was proposed. The method furnished good results provided the geometry factor of the pressure drop relationship for laminar flow was known.

The classical work of Trupp and Azad(1975) discussed the structure of the flow through a wind tunnel study of a fully turbulent flow for three tube spacing (P/D = 1.2, 1.35 and 1.5) over the Reynolds number range of 12000–84000. Measurements were made for local wall shear stresses, the distribution of mean axial velocity, Reynolds stresses and eddy diffusivities.

Trying to develop a more general approach to the problem, Carajileskov and Todreas(1976) applied an one-equation statistical model of turbulence to an interior subchannel of a bare rod bundle. The velocity field was detailed described identifying axial and secondary flows. The wall shear stress for several aspect ratios of P/D was also computed. The results were compared with their own experiment; the velocity distribution, turbulent kinetic energy and Reynolds stresses were measured through a laser Doppler anemometer. Water was the working fluid. This work was soon followed by a two-equation anisotropic turbulence modelling of the flow (Bartzis and Todreas, 1979). Heat transfer predictions were also made in this work to confirm the importance of anisotropic viscosity in temperature predictions; these predictions, however, were not compared with any experimental data. At nearly the same time, a second work was released by Fakory and Todreas(1979) which main objective was to study the wall shear stress distribution around the rod periphery; static pressure distribution, turbulence intensity and the friction factor in the central subchannel for Reynolds number between 4000 to 36000 were also investigated; the working fluid was air. The results showed that the maximum wall shear stress occurs at the largest flow area and that the static pressure is not uniform around the rod periphery.

These studies about the hexagonal geometry were soon followed by others where authors were concerned with square arrays (see Marek et al.,1973), Hooper(1980), Hooper and Wood(1984), Hooper and Rehme(1984), Renksizbulut and Hadaller(1986)). In fact, many works were available before. However these concentrated mainly on global results related to the pressure drop and the transfer of heat.

The work of Marek et al.(1973) covered results of pressure drop and heat transfer on two rod bundles with smooth surfaces; also pressure losses at the spacers and the circumferencial temperature distribution were measured.

Hooper(1980) was mainly concerned with the measurement of mass transfer between adjacent flow subchannels. The results were expressed as gross diffusion coefficients according to every simulated geometry. The work also provided a detailed account of the mean velocity variation, the wall shear stress variation and the six terms of the symmetrical Reynolds stress tensor. The turbulent flow structure was shown to depend strongly on the P/D ratio. In a follow up paper, Hopper and Wood(1984) used the axial momentum integral equation to show that the shear stress distribution is primarily determined by the pressure gradient and the shear stress $\overline{uv}$, a result that confirms the negligible influence of the mean secondary flow on flow parameters. In the same year, Hooper and Rehme(1984) showed that the axial and azimuthal turbulence intensities in the rod gap region increased strongly with decreasing rod spacing. The existence of features that promote intersubchannel momentum and heat transfer reduce the azimuthal variation of both the skin-friction coefficient and the heat transfer.

The distribution of wall shear stress, mean axial velocities and turbulence intensities for the Reynolds number 500000 were measured by Renksizbulut and Hadaller(1986) through the laser Doppler anemometry and calibrated Preston tubes.

We conclude this section pointing out to the reader that all mentioned works, no matter how complete they are, had to settle for a particular geometry when performing the measurements or computations. Because of this, the proposition of general results for flows with such a degree of complexity has always been a problem. In the next section we will extend the procedure of Lee(1995) to the thermal field and to the case of a rough surface yielding a procedure which, despite its simplicity, covers most situations of interest as far as the design of nuclear reactors is concerned.

3. THEORY

As vastly known, the law of the wall provides a useful way to describe the near wall velocity and temperature profiles. The simplicity of the law of the wall formulation, allied to its robustness, has decisively contributed to make its use very popular over the years.

The law of the wall, as written by most authors, assumes the form

$$\frac{u}{u_\tau} = \frac{1}{\varkappa} \ln \frac{y u_\tau}{\nu} + B, \tag{1}$$

where, $\varkappa = 0.4$ and $B = 5.0$ are universal constants.

This expression is expected to hold for two-dimensional, incompressible, attached flow over a smooth surface.

Of course, many specialisations of the law of the wall have been proposed in literature to account for such diverse effects as surface roughness, transpiration, pressure gradients, three-dimensionality, compressibility and shock-wave interaction, among many others. In fact, we could spend a long time here reviewing all aspects related to different specific problems and their implication on the law of the wall. However, since the major concern of our work is with flows over rough surfaces, we will refrain ourselves by just introducing the formulation of Clauser(1956) for pipe flow, for the average roughness height denoted by k

$$\frac{u}{u_\tau} = \frac{1}{\varkappa} \ln \frac{y u_\tau}{\nu} + B - \Delta B(k^+), \tag{2}$$

$$\Delta B_{\text{sand grains}} = \frac{1}{\varkappa} \ln \left(1 + 0.3k^+\right), \tag{3}$$

Three roughness regimes can be defined by using of $k^+ = k u_\tau/\nu$, i.e, hydraulically smooth wall for $k^+ < 4$, transitional-roughness regime for $4 < k^+ < 60$, and fully rough flow $k^+ > 60$. In the last case, the Clauser's law of wall, Eq. (2) and (3) reduces to that proposed by Nikuradse(1933).

Investigating the flow in sand-roughened pipes, Nikuradse(1933) established that, at high Reynolds number, the near wall flow becomes independent of viscosity, being a function of the roughness scale, of the pipe diameter and of Reynolds number. He also found that, for the defect layer, the universal laws apply to the bulk of the flow irrespective of the conditions at the wall. The roughness effects are, therefore, restricted to a thin wall layer.

Having said that, we must now acknowledge that it is extremely difficult to propose universal relations for the velocity field owing to the large number of parameters necessary to characterise the roughness. Again, we consider this not to be the place for a long discussion on theories about turbulent flow over rough surfaces so that the developments will be kept to a basic standard.

Basically, Nikuradse showed that the law of the wall can be re-written in terms of a length scale characteristic of the roughness,

$$\frac{u}{u_\tau} = \frac{1}{\varkappa} \ln \frac{y}{k_s} + C, \tag{4}$$

where, k_s denotes this length and C assumes different values depending on the type of rough surface.

The law of friction in pipes can now be deducted considering that the law of the wall applies over the entire pipe radius. Integration of the classical law of the wall, Eq. (1) results in

$$\left(\frac{2}{f}\right)^{1/2} = \frac{1}{\varkappa} \ln\left[Re\left(\frac{2}{f}\right)^{1/2}\right] + B'. \tag{5}$$

where f denotes the friction coefficient, Re the Reynolds number based on the pipe diameter and B'=1.57.

For the typical geometry of an infinite rod bundle with an infinity number of rods the mean velocity can evaluated through the integration

$$\overline{u} = \frac{1}{A} \int u \, dA = \frac{1}{A} \int_0^{\theta_{max}} \int_0^{(P/2\, sec\theta - R)} (R + y) u(y) dy d\theta, \tag{6}$$

where

$$A = \int_0^{\theta_{max}} \frac{1}{2}\Big(\frac{P}{2}sec\theta - R\Big)\Big(\frac{P}{2}sec\theta + R\Big)d\theta, \tag{7}$$

denotes the channel area where flow symmetry is supposed to hold; P denotes the centerline distance between the rods, the pitch, and R the radius of the rods. The distance y is taken from the periphery of the rods out in radial direction; the angle θ_{max} measured in radians defines the type of symmetry considered, whether a triangular or square array.

If we now consider that the law of the wall applies to all the flow region, Eqs. (1) or (2) could be substituted into Eq. (6) to find the friction factor equation. However, we have seen in the previous section that the wall shear stress varies along the rod periphery, and, because of this, the integration cannot be performed unless this angular variation is first modelled. The common assumption in this case is to take $u_\tau = F(\theta)\overline{u_\tau}$. Then, appealing to the data of Trupp and Azad(1975) and of Fakory and Todreas(1979) for the hexagonal array and of Hooper(1980) and of Renksizbulut and Hadaller(1986) for the square array, it is possible to write

$$\frac{u_\tau}{\overline{u_\tau}} = \Big(\frac{2}{f}\Big)^{1/2} = (1 - a\,\cos\,6\theta - b\,\cos\,12\theta)^{1/2}, \qquad \text{hexagonal array} \tag{8}$$

$$\frac{u_\tau}{\overline{u_\tau}} = \Big(\frac{2}{f}\Big)^{1/2} = (1 - a\,\cos\,4\theta - b\,\cos\,8\theta)^{1/2}, \qquad \text{square array} \tag{9}$$

where the constants a and b must be determined through a data fitting; the resulting values are shown in Table 1.

Tabela 1: Constants a and b.

Hexagonal array			
P/D	1.11	1.15	1.19
a	0.25	0.12	0.02
b	0.02	0.02	0.01
Square array			
P/D	1.11	1.15	1.19
a	0.20	0.09	0.07
b	0.02	0.02	0.02

The substitution of Eqs. (8) and (9) into Eq. (6) results in

$$\begin{aligned}\frac{\overline{u}}{\overline{u_\tau}} &= \Big(\frac{1}{\varkappa}\ln\frac{\overline{u_\tau}}{\nu} + B\Big)\frac{1}{A}\int_0^{\theta_{max}}\frac{1}{2}F(\theta)\Big(\frac{P}{2}sec\,\theta - R\Big)\Big(\frac{P}{2}sec\,\theta + R\Big)d\theta \\ &+ \frac{1}{A}\int_0^{\theta_{max}}\Big[\Big(-R\,F(\theta)\Big(\frac{P}{2}\sec\theta - R\Big)\Big) - \frac{1}{4}F(\theta)\Big(\frac{P}{2}\sec\theta - R\Big)^2 \\ &+ \frac{F(\theta)}{2}\ln\Big[F(\theta)\Big(\frac{P}{2}\sec\theta - R\Big)\Big]\Big(\frac{P}{2}\sec\theta - R\Big)\Big(\frac{P}{2}\sec\theta + R\Big)\Big]d\theta.\end{aligned} \tag{10}$$

This equation is not new, having been presented earlier in Lee's paper; it defines an algebraic transcendental function that can be solved to give values of u_τ. The friction factor then follows immediately.

Specialisation of the above equation to rough surfaces is quite simple consisting basically in integrating Eq. (2) instead of Eq. (1). As mentioned earlier, Eq. (4) is included in Eq. (2). The procedure is quite straightforward and for this reason will not be repeated here.

All the above ideas can be applied to the temperature field, provided the thermal law of the wall is considered together with the concept of bulk temperature.

The thermal law of the wall for flow over a smooth surface can be written as

$$\frac{T_w - t}{t_\tau} = \frac{1}{\varkappa_t} \ln \frac{y u_\tau}{\nu} + B_t(Pr), \tag{11}$$

where, t_τ is the friction temperature, T_w is the wall temperature, $\varkappa_t$ (=0.44), and B_t is a function of the Prandtl number, Pr

$$B_t(Pr) = 13 Pr^{2/3} - 7 \tag{12}$$

The bulk temperature of the flow is defined by

$$\overline{t} = \frac{\int t\, u\, dA}{\int u\, dA} = \frac{1}{\overline{u}\, A} \int_0^{\theta_{max}} \int_0^{(P/2\, sec\theta - R)} (R + y) u(y)\, t(y) dy d\theta, \tag{13}$$

Similarly to the wall shear stress behaviour, we expect the wall heat transfer coefficient to vary along the rod periphery. Thus, if somehow this variation can be accounted for, then Eqs. (11) and (13) can be used to give predictions of the heat transfer coefficient.

Here, invoking the existing similarity between the transport mechanisms in the velocity and temperature fields for turbulent flows we make

$$t_\tau = \overline{t_\tau}\, H(\theta), \qquad H(\theta) = F(\theta). \tag{14}$$

Then, it results from Eqs. (2), (11) and (13), that

$$\begin{aligned}
\frac{\overline{u}\,\overline{t}}{\overline{u_\tau}\,\overline{t_\tau}} = \frac{1}{A} \int_0^{\theta_{max}} \Big[& 2\, L(\theta)\, R\, \xi(\theta) + \frac{1}{4} L(\theta)\, \xi(\theta)^2 - \frac{1}{2} L(\theta)\, \xi(\theta)\, \left(4\, R + \xi(\theta)\right)\, \ln(\xi(\theta)) \\
& + \frac{1}{2} L(\theta)\, \xi(\theta)\, \left(2\, R + \xi(\theta)\right)\, \ln^2(\xi(\theta)) \\
& - \left(L(\theta)\, \left(N(\theta) + M(\theta)\right)\, R\, \xi(\theta)\right) - \frac{1}{4} L(\theta)\, \left(N(\theta) + M(\theta)\right)\, \xi(\theta)^2 \\
& + \frac{1}{2} L(\theta)\, \left(N(\theta) + M(\theta)\right)\, \xi(\theta)\, \left(2\, R + \xi(\theta)\right)\, \ln(\xi(\theta)) \\
& + L(\theta)\, N(\theta)\, M(\theta)\, R\, \xi(\theta) + \frac{1}{2} L(\theta)\, N(\theta)\, M(\theta)\, \xi(\theta)^2 \Big] d\theta;
\end{aligned} \tag{15}$$

where the following definitions have been used

$$\xi(\theta) = \frac{P}{2}\,\sec(\theta) - R, \tag{16}$$

$$L(\theta) = \frac{1}{\varkappa\varkappa_t}\,F(\theta)\,H(\theta), \tag{17}$$

$$M(\theta) = \ln\,F(\theta)\,\frac{\overline{u_\tau}}{\nu} + B, \tag{18}$$

$$N(\theta) = \ln\,F(\theta)\,\frac{\overline{u_\tau}}{\nu} + B_t. \tag{19}$$

The Equation (15) can now be explicitly evaluated solved to obtain the heat transfer coefficient $C_h(=\overline{u_\tau}\,\overline{t_\tau}/\overline{u}\,\overline{t})$, once the friction velocity $\overline{u_\tau}$ is obtained from the solution of Eq. (10).

The above results can immediately be extended to rough surfaces only if the forms of Eq. (18) is changed to Eqs. (2).

4. RESULTS

The transcendental Equation (10) is solved to obtain the friction factor, f, as a function of the Reynolds number, $Re(=\rho\overline{u}D_e/\mu)$, the pitch-to-diameter ratio P/D, and the average roughness height k/D, where D_e is the hydraulic diameter of the subchannel passage. A typical result is shown in Fig. 1. We can observe that the surface roughness increases considerably the friction factor. For the roughness heights considered, $k/D = 0.001, 0.002, 0.005$, k^+ has the value of 43, 113, and 312, ranged from transitional roughness to fully rough regime. A typical result of heat transfer coefficient, C_h, obtained from Eq. (15), is shown in Fig. 2. We see that surface roughness enhances the turbulent heat in rod bundle, but less in percentage when compared with the increase in friction factor. In Fig.3 we shown the ratio of the heat transfer coefficient to that of a smooth circular pipe, C_{hs}, for a square array of rod bundle. We can notice that for a same surface roughness, the heat transfer enhancement varies with P/D and increases with increased Reynolds number. In Fig.4, we can verify the departure of Reynolds analogy ($C_h = f/2$) with various surface roughness. With smooth surface, the Reynolds analogy is seen to be maintained, while for fully rough surface, $k/D = 0.005$, C_h is nearly half of $f/2$. From an engineering point of view, it would be interesting under some circumstance to adopt rod bundle heat exchanger with rough surface if it is worthy to pay the overhead in pressure drop.

5. CONCLUSION

The present work has shown how the method of Lee(1995) can easily be extended to the prediction of turbulent flows over rough surfaces. The work has also shown how all developments can be applied to the temperature field, providing a useful relation for the prediction of the heat transfer coefficient. This equation, Eq. (15), has been proposed here

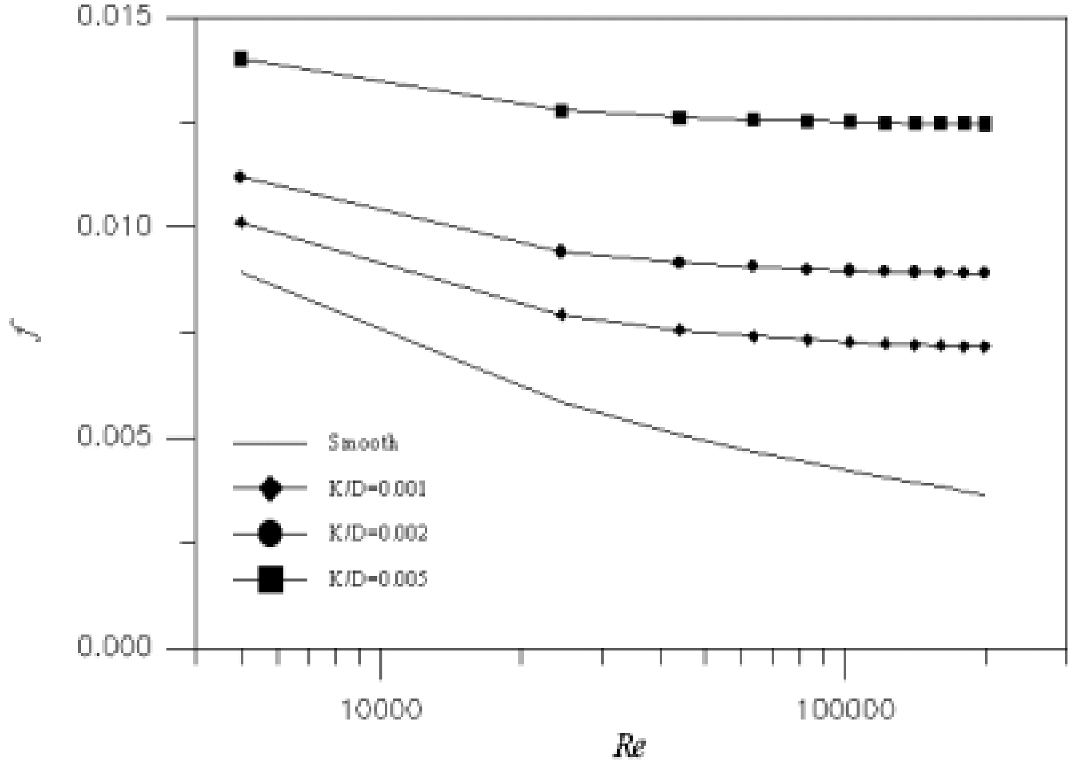

Fig.1 Friction factor for an hexagonal array, $P/D = 1.05$

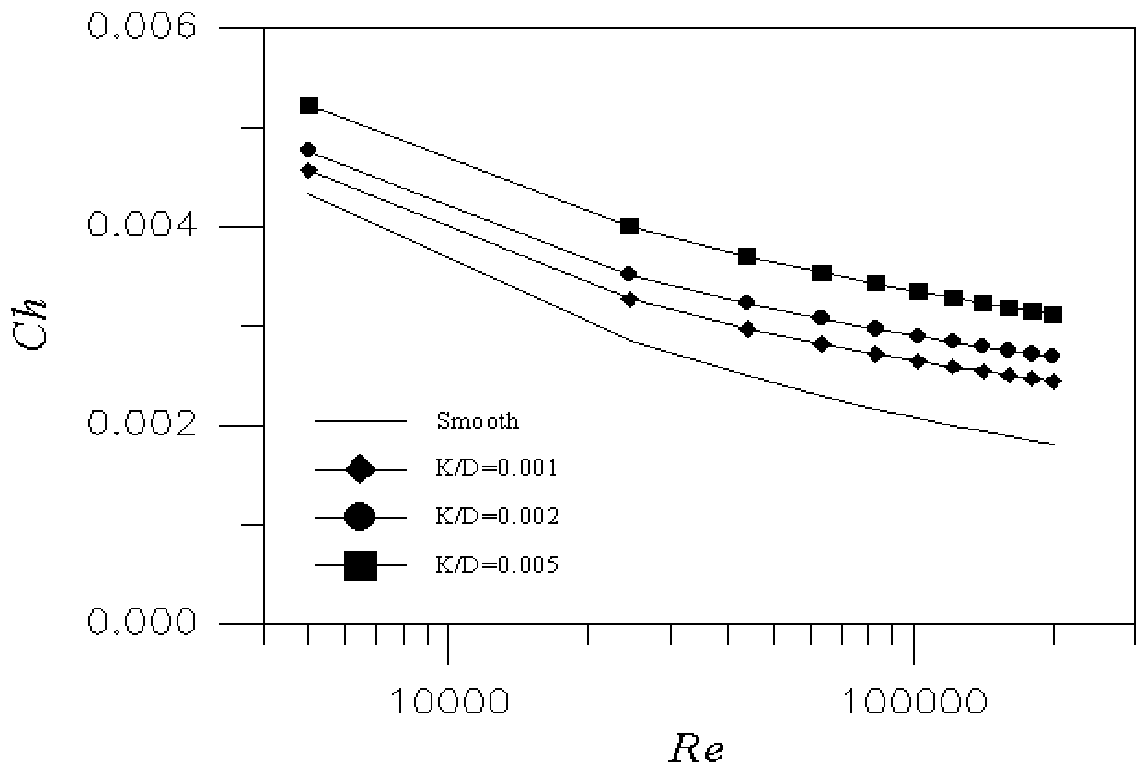

Fig.2 Heat transfer coefficient for an hexagonal array, $P/D = 1.05$ and $Pr = 0.9$

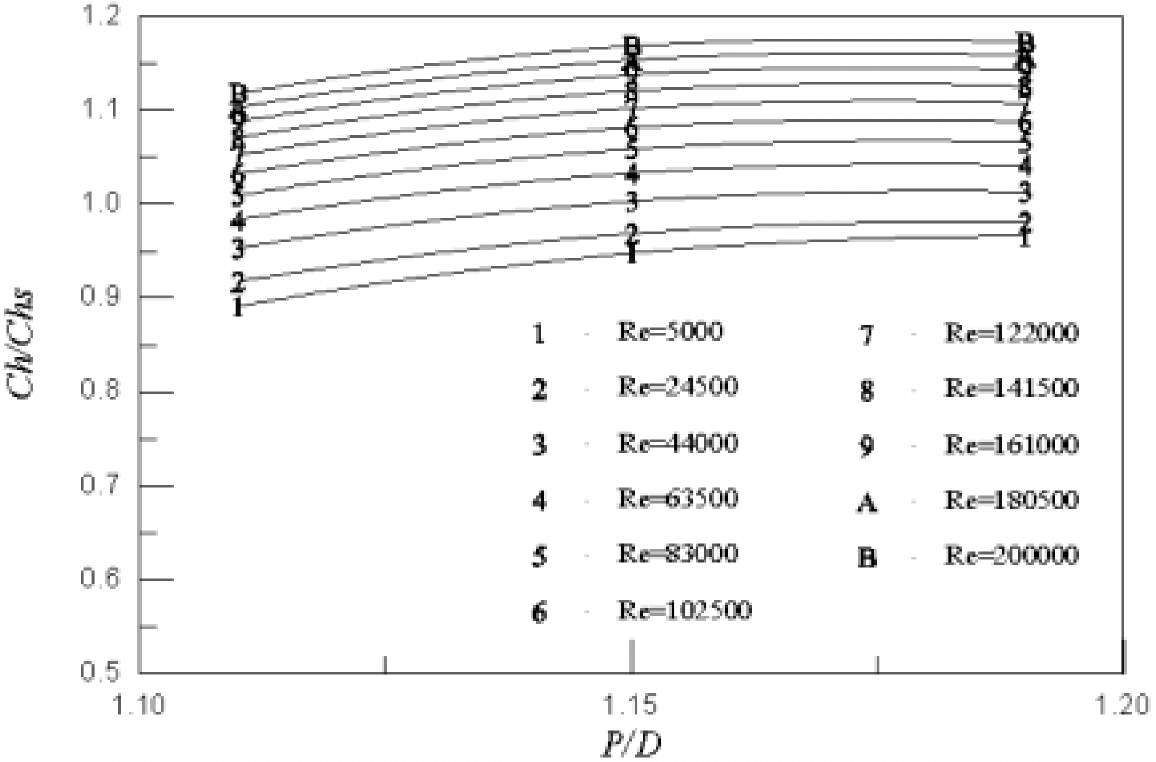

Fig.3 Influence of pitch-to-diameter ratio and Reynolds number in heat transfer coefficient
Square array, $k/D = 0.001$ and $Pr = 0.9$

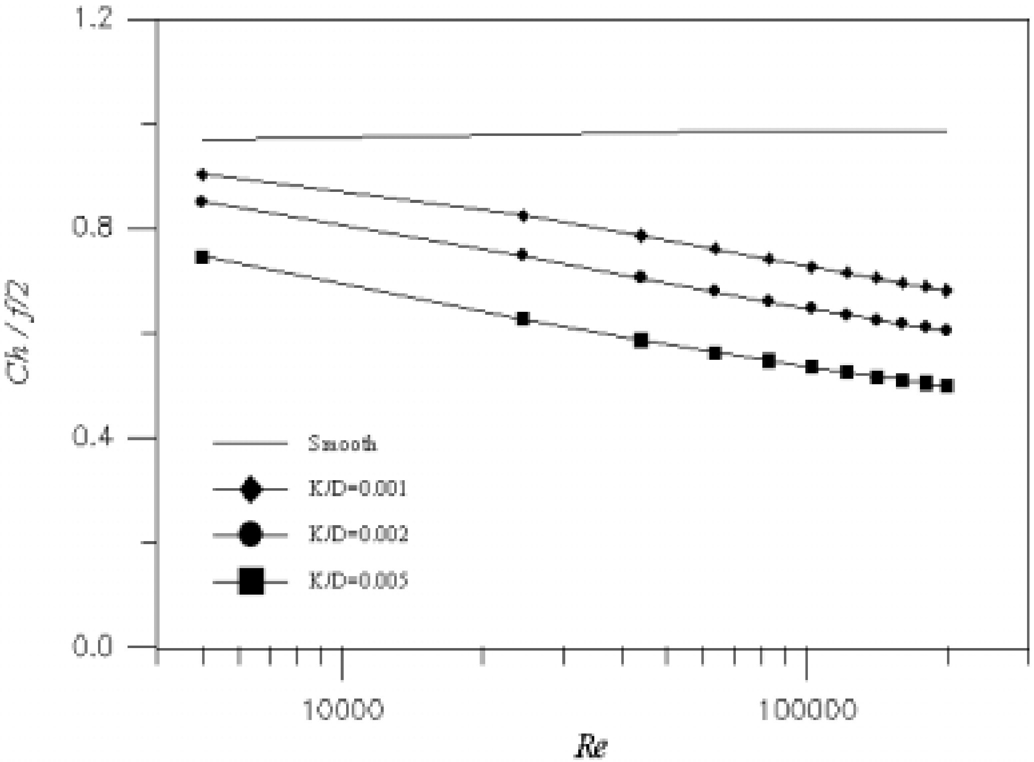

Fig.4 Departure from Reynolds analogy
Hexagonal array, $P/D = 1.05$ and $Pr = 0.9$

for the first time. Numerical computations for several flow geometries have shown that the theory is plausible, constituting a serviceable engineering tool.

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